A Mixed Model for Real-Time, Interactive Simulation of a Cable Passing Through Several Pulleys

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Abstract. A model of a cable and pulleys is presented that can be used in Real Time Computer Graphics applications. The model is formulated by the coupling of a damped spring and a variable coefficient wave equation, and can be integrated in more complex mechanical models of lift systems, such as cranes, elevators, etc. with a high degree of interactivity.

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INTRODUCTION

Physically based modelling has become, specially in the last years, one of the key subjects in many Real-Time Computer Graphics (RTCG) applications; simulation for training, video-games, Virtual Reality learning applications,... all of them share the need of a simulated environment, in which the user is immersed. Every mechanical or logical system involved in the simulation has to be properly modeled, in order to maximise the feeling of *presence* (the feeling of *being there*), and to achieve the particular goals of the application; e.g. in a driving simulator for training, the vehicle model has to be accurate enough so that the necessary driving skills can be acquired by the subject.

However, better results are not always reached with a more exact or realistic model; beyond certain level of exactness, the difference between two different models cannot be appreciated by the user. Other aspects, such as a proper geometric modelling of the environment, or an adequate and constant frame rate can make the difference between a good application and a useless system. Indeed, the high computational cost of the graphic representation of the scene, often leaves a narrow margin in the dynamic models that, in addition, have to be simulated in real time. For this reasons, a low computational cost is a must for any model that has to be used for RTCG applications.

Another characteristic of RTCG applications is their complexity, involving models of very different kind of systems, and their high degree of interactivity. Thus, the capability of a certain model to be integrated in an interactive manner within a complex application, and its flexibility to simulate a wide range of situations, are usually very appreciated features, rather than a very accurate model.

Within the field of simulation for training, crane simulators have become a useful tool in many industry fields, as training is dangerous and expensive, and also because many situations can be reproduced in a simulator and not with a real crane, for safety reasons. Such simulators involve complex mechanical systems of the machinery and of the loads, but one of the aspects most poorly modeled so far is pulley and cable systems, used in most cranes.

Pulley and cable dynamics

Cable dynamics has been extensively studied from different modelling methodologies [1, 2, 3] leading, in most cases, to non-linear partial differential equations or to finite element schemes. These models, however, are addressed to a good understanding of the problem or to an accurate simulation, useful for structure analysis or design, and the associated numerical models are computationally intensive and not adequate for RTCG applications. Also, a common problem not often considered [4], is the pay-out/reel-in process, that is the basis of load lift and boom movement.

By the other side, cable-pulley models can be found as elements for finite element analysis [5, 6, 7], or in analysis of traction systems such as elevators or transport belts [8, 9]. But, as in the previous case, models found in the bibliography are computationally to expensive to our needs. Moreover, they usually do not consider the dynamics of the cable between two pulleys.

We present a computationally efficient model for a cable and a set of pulleys attached to rigid bodies. The model considers both the forces that appear on these bodies, and the oscillatory movement of the cable segments, as well as the pay-out/reel-in process. Also, it allows an straightforward integration in complex simulation environments, with the possibility of collision with other bodies in the environment.

The rest of the paper is organised as follows. First a description of the system, based upon a damped spring is presented, together with some notation. Then, the wave equation with variable coefficients is proposed for the simulation of the oscillation of the cable between pulleys. Later, the two models are coupled, in order to obtain a unique model, paying attention to some implementation issues.

CABLE AND PULLEYS MODEL

In this section we describe the model that provides the tension of the cable from the position of every pulley. It allows the dynamic interaction of the cable-pulley system with the rest of the simulated environment.

The cable is first considered as a force element that acts on the bodies linked to it. A cable suspended by its ends is considered as a massless, damped spring. The stiffness constant of the spring is given by the young modulus E of the cable and by its length L, as $k = E/L$. Thus, two forces will appear at both ends of the cable if the distance between them l is grater than the original (undeformed) length of the cable L. This force, that we will call tension, is given by

$$T = \begin{cases} \max\left(\frac{E}{L}(l-L) - c(l_t - L_t), 0\right) & \text{if } l > L \\ 0 & \text{otherwise.} \end{cases} \qquad (1)$$

where c is a damping constant, which depends on the properties of the cable. Here L_t represents the rate of shortening or lengthening of the cable per unit time, considering the pay-out/reel-in process, and l_t the change rate of the distance l, per unit time. The maximum, applied at (1), bans the possibility that repulsion forces might appear due to the damping term. According to this model, no variation in the tension is possible along the cable, avoiding the related calculus and preventing from possible instabilities due to high frequency modes.

Now let us address a cable that passes through a set of points, that represent pulleys attached to the world or to some bodies. The pulleys are considered massless and frictionless, allowing free movement of the cable along them, and tension at both sides of the pulley to be the same [2]. Let us consider a set of nodes $P_0, \ldots, P_N$, everyone of them possibly attached to a body. We shall denote the Euclidean distance between nodes i and $i+1$ as $l_i = |\vec{d_i}| = |P_{i+1} - P_i|$ for $i = 0, \ldots, N-1$. Let us make a cable pass through this succession of points; every node is considered as a pulley, while nodes 0 and N are considered as the points where both ends of the cable are rigidly linked.

Just applying the scheme presented above, the tension is obtained by calculating the difference between the length of the cable L and the sum of the distances of neighbour nodes $l = \sum l_i$. Once the tension is obtained, it can be applied to every body that is attached to one of the pulleys of the system.

Before every step of the simulated world, distances l_i, and the current length of the cable $l = \sum l_i$ together with their change rates, are calculated. Then, tension T is calculated, using (1). At every point P_i attached to a body, apply forces $F_i^- = -T\vec{d}_{i-1}/l_{i-1}$ and $F_i^+ = T\vec{d_i}/l_i$, except at nodes P_0 and P_N where only F_0^+ and F_{N-1}^- are applied, respectively.

CABLE OSCILLATION MODEL

In the model depicted so far mass is deprecated, and thus no inertia nor oscillation of a cable segment are considered. This has two negative effects in the simulation; the bodies attached to the cable are not affected at all by these phenomena, and the visual quality of the simulation is reduced. To overcome these drawbacks a model based on the wave equation will be coupled with the spring model in order to consider cable oscillations.

The wave equation with variable coefficients

The evolution of an elastic string, suspended by its ends, damped, and under the action of an external force $F(x,t)$, is given by the partial differential equation [10]

$$u_{tt} + cu_t = a(t)^2 u_{xx} + F(x,t); \qquad u(0,t) = g(t); \qquad u(l,t) = h(t), \qquad (2)$$

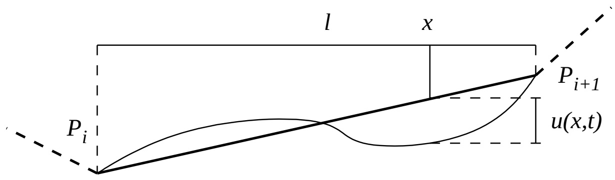

FIGURE 1. Variables of the oscillation model

where $u(x,t)$ is the distance of point x on the string, from the segment that joins both ends, at time t. Function $a(t)$ represents the propagation speed of a pulse along the string at time t, and depends on the string tension $T(t)$ and density ρ by the relationship $a^2 = T/\rho$. The wave equation with variable coefficients has been studied by many authors, both analytically and numerically [11, 12, 13]. Equation (2) can be scaled by means of the change of variable $ls = x$, leading to an equivalent model in the unit interval:

$$u_{tt} + cu_t = \frac{a^2}{l^2} u_{ss} + F(ls,t); \qquad u(0,t) = g(t); \qquad u(1,t) = h(t). \tag{3}$$

Oscillating cable

Let us consider the cable segment that goes from point P_i to point P_{i+1}. Now the oscillation of the points of the cable is to be considered. In order to clarify the presentation, the 2-dimensional case will be considered, $P_i = \left(P_i^1, P_i^2\right)\right) \in \mathscr{R}^2$

Under the previously stated assumptions of constant tension and density across the cable, let us consider a point at the cable between P_i and P_{i+1}, and let x be the horizontal distance of the point from P_i. The transversal deviation of this point will be denoted as $u(x,t)$ (see Figure 1). Using the scaled equation (3), the evolution of $u(x,t)$ can be modeled by

$$u_{tt} + cu_t = \frac{T}{\rho (l_i^1)^2} u_{ss} + F(ls,t); \qquad u(0,t) = P_i^2(t); \qquad u(1,t) = P_{i+1}^2(t). \tag{4}$$

This model is a variable coefficient equation, as both the tension of the cable T and the horizontal distance between the nodes $l_i^1 = |P_{i+1}^1 - P_i^1|$ are time dependent.

At this point a remark is necessary in order to justify the use of equation (4); the time dependence of l_i, makes that in the variable change $ls = x$, time derivatives of $u(s,t)$ involve partial derivatives respect to the spatial variable s, as a result of the chain rule application. However, equation (4) is the equation chosen for the model, neglecting such terms. In order to understand this reason, a note on the magnitudes of ρ, T and l is necessary.

A steel cable of the type used in a tower crane has a density of 1-2kg/m, and usually works under a tension between 5000N and 50000N (higher in the case of heavy load cranes), with lengths between pulleys of 5-50m. If these numbers are introduced in the equation, the neglected terms are several orders of magnitude smaller than those in equation (4).

So, even though the used model is less accurate when cable length is varying, the nonlinear wave equation (4) still gives a behaviour that is within the needs of our application with less computational cost, and has the important advantage that finite difference schemes inherit the stability properties of the constant coefficient version [13].

COUPLING OF THE MODELS AND DISCUSSION

Two models have been presented so far, for the dynamic simulation of a cable and pulleys system; the one for the global tension, that provides the means for interacting with the environment (application of forces), and the other, that simulates the movement of the cable between nodes. Now a coupling between the models will be proposed, in order to develop a unique model that considers both behaviours.

The first model proposed in this work takes as its input the actual (deformed) length of the cable, calculated from the position of the nodes. This input is given by the evolution of the environment by the evolution of these positions. Then a tension is computed, that provides the forces that form the output, influencing the dynamics of the environment. The second model takes as input the tension of the cable and the length of each segment, and provides the transversal displacement of the cable at every point.

The output of the first model (the tension of the cable) is the input of the wave equation model. In order to close the loop, the calculation of the input of the spring model to take into account the shape of each cable segment; the deformed length of the cable will be the sum of the arc lengths of every cable segment, according to the wave equation model. Using an explicit finite difference scheme for the solution of the wave equation, the algorithm can be depicted as follows:

1. Using last value of T and l_i, evolve one step of the finite difference scheme for the wave equation,
2. Calculate an approximation to the arc length of every segment $\hat{l}_i$, and current cable length $l = \sum \hat{l}_i$
3. Calculate the new value of tension T using formula (1),
4. At every point P_i, apply forces $F_i^- = -T\vec{d}_{i-1}/l_{i-1}$ and $F_i^+ = T\vec{d}_i/l_i$, except at nodes P_0 and P_N where only F_0^+ and F_{N-1}^- are applied, respectively.

Simulation Results

The proposed model has been implemented and integrated in two crane simulators; a Tower Crane simulator and a Rubber Tyred Gantry Crane simulator. Some efficiency comparisons have been done, with a multibody chain model similar to the one presented in [4]. The results are highly satisfactory, as the computational cost of the proposed scheme is between one tenth and one fiftieth of the cost of the multibody approach using the augmented formulation approach. Indeed, the cost of the multibody approach provoked the loss of Real-Time once the total number of chain nodes was between 25 and 30, with severe instability problems in many near-singular configurations.

CONCLUSION

A cable and pulleys system dynamics model has been presented and explained. A combination of a simple spring model together with a variable coefficient wave equation leads to a straightforward computational scheme, for the simulation of cable-pulleys systems. This model covers the main needs of real-time computer graphics simulations; low computational cost and the capability of performing interactive simulations in complex simulation environments.

The model is integrated in a very straightforward manner in mechanical models of cranes, bridges, etc. for its output is a set of forces applied at given points. Collision detection, together with a dynamic node insertion and deletion procedure, can be easily integrated, allowing for full interactivity. No cable model has been found so far in the bibliography, with such capabilities for real time interactive simulation.

As a continuation of this work, a more in depth evaluation of the neglected terms in the wave equation with variable length has to be done. Also, the possibility of using different finite difference schemes, seeking for efficiency and stability has to be evaluated.

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